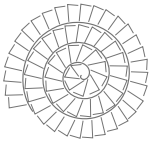

Mathematical Staircase, Inc.

MathILy-EST

MathILy-EST 2019 Final Report

Preface

The MathILy-EST Research Experience for Undergraduates (REU) was created to serve college-age students that are early in their college career (i.e., an emphasis on freshmen, but with consideration for sophomores and even graduating high-school seniors). Also, this REU runs in parallel to the 5-week MathILy program for talented high-school students, both taking place at Bryn Mawr College and with all the students and staff sharing the same dorm building.

2019 was the first year of NSF funding for MathILy-EST (DMS-1851842) and the first year of the program.

Program Preparations

Promotions

Electronic: Emails announcing MathILy-EST were sent to the {MathILy, MathILy-Er} mailing list, to multiple email lists in the professional math community, to departments and individuals at Historically Black Colleges and Universities and Minority Serving Institutions, and to other contacts in order to spread word of this new program. Web pages for MathILy-EST were created and added to online lists of REU programs.

Webpages and links: MathILy-EST has its own webpages, and of course it is listed on the NSF-REU pages. MathILy-EST was added to the AMS REU and Opportunities pages. The Minion submitted an entry to the Institute for Broadening Participation's pathwaystoscience.org, requested that MathILy-EST be listed as a friendly REU at the Math Alliance website, and created an entry in the Art of Problem Solving's wiki.

Website traffic: There were 1,400 impressions for the mathilyest/index.html page in March, 3,300 in April, and 850 in May. The facts page got 1700 hits in April, and the applications page got 1600 hits in April. The only recognizable referrer was the AMS REU list, which produced about 50 hits in March and 80 in April.

Other Activities: sarah-marie and Tom attended a Discrete Mathematics Day at UMass, Amherst where they announced the REU. Personal contacts of Tom's promoted the REU to their social media presences. sarah-marie offered an Art of Problem Solving Math Jam on REUs in general and MathILy-EST in specific, with a Q&A (~40 attendees for most of the session). sarah-marie described MathILy-EST at the end of her Penn State ARML talk (~250 attendees) and one student asked later to be notified when applications opened for 2020.

Applications and Admissions

Statistics: There were 112 applications for the 6 REU slots.

Demographics: Applicants originated from 27 US states (about 44 were from CA, PA, MA, and NY). Of course, some states were over-represented in that number, with at least 13 students from CA schools, 11 from PA, 10 from MA, and 10 from NY. The small (for an REU) number of applications is almost certainly because of the late announcement of the program in the REU application season, and this likely contributed to only 27 states being represented in the applicant pool.

The data in the following table was mostly self-reported by the applicants.

Stage in application	Female	Asian-American	African-American	Latinx	SLAC
All applicants	38%	17%	5%	4%	26%
First cut (34)	50%	18%	15%	6%	29%
Second cut (13)	54%	23%	8%	15%	42%
Invited	50%	50%	16%	16%	33%

Every student invited to participate accepted, and all within 48 hours of invitation.

Personnel

Administrative: The MathILy-EST 2019 Director was Dr. Thomas Hull (mathematics faculty at Western New England University). The PI on the NSF grant was sarah-marie belcastro (President of Mathematical Staircase, Inc.). The {MathILy, MathILy-Er, MathILy-EST} Minion was Madison Stuart.

Senior Personnel: These individuals gave advice on the construction of MathILy-EST and the NSF proposal for the grant that funds the program.
Hannah Alpert, NSF Postdoctoral Fellow at the University of British Columbia, Canada
Max Engelstein, mathematics faculty at the University of Minnesota
Nate Harman, NSF Postdoctoral Fellow and Dickson Instructor at the University of Chicago
Peter Tingley, mathematics faculty at Loyola University Chicago

What Happened at MathILy-EST 2019?

Academics/Research

Before participant arrival on site: The first week of MathILy-EST was off-site. Participants were each sent a different research paper on the REU topic to read, with video conference meetings with the Director and each other to help understand the readings.

On site: Weeks 2–7 of MathILy-EST took place at Bryn Mawr College. After presenting the topics of their reading papers to each other, the Director gave open research problems to the students. The students formed two groups of three, where each group focused on a different problem. One group chose to work on finding a general method to relate counting the number of locally-valid mountain-valley (MV) assignments of a flat origami crease pattern to counting the number of ways to properly 3-vertex-color a corresponding graph. The other group chose to explore origami flip graphs, a new

concept encoding the relationship between valid MV assignments of a given flat-foldable crease pattern with a graph (the origami flip graph) whose vertices are all (locally) valid MV assignments and where two MV assignments μ1 and μ2 are connected by an edge if and only if one can "flip" a single face to turn μ1 into μ2 (a face "flip" means to reverse the mountains and valleys along the creases bordering that face).

Both groups made a lot of progress and achieved publishable results during the program. The graph coloring group generated a large family of graphs whose proper 3-colorings are in bijection with valid MV assignments of single-vertex flat-foldable crease patterns. They also proved that given a multiple-vertex flat-foldable crease pattern C, the 3-colorable graphs for the individual vertices of C can tile to make a larger graph whose 3-colorings correspond to locally-valid MV assignments of C. This establishes very strong evidence that the combinatorial structure governing locally valid MV assignments of flat-foldable crease patterns is the 3-coloring of graphs. The origami flip graph group proved a complicated theorem that classifies when the origami flip graph of a single-vertex crease pattern is connected. They also looked at the extreme cases of single-vertex crease patterns. Those with the fewest number of valid MV assignments (for a fixed vertex degree $2n$) have origami flip graphs that are subgraphs of the n-cube graph. Those with the largest number of valid MV assignments have origami flip graphs with diameter n and that are otherwise very complicated; even calculating the number of edges requires an algorithm, which the group implemented in Python, that employs integer partitions and Burnside's Theorem.

At the end of the seventh week, when the MathILy-EST students departed for home, the groups had extensive drafts of their papers written, but both needed a lot of work in terms of finishing proofs and getting the writing to a journal-submittable level. And both groups had made plans for getting things done during the last, off-site week of the program.

Virtual work after departure from site: Off-site, the students made substantial progress but did not complete their papers by the end of the program. The coloring group was close to being done, while the origami flip graph group still had two major proofs unfinished.

On October 4, 2019 the coloring group's paper was posted to the arXiv. Also by this time, the origami flip graph group had completed some of their remaining proofs, but still had unfinished details. The group decided to split their work into two papers, one focusing on the when the origami flip graph is connected and the other focusing on the single-vertex all-equal-angles case. A major re-writing will be needed to re-organize this work into two papers. All told, the Director finds that the work of each group is an excellent contribution to the combinatorial geometry studies of flat origami crease patterns.

Professional Development

Presentations: The MathILy-EST 2019 program required the participants to make many presentations. Every time that MathILy had a visitor to give a Daily Gather, the visitor also met with the MathILy-EST students, whereupon the students presented their research topics and work-to-date to the visitor. During week 6 of the program, the Director and MathILy-EST students took the train to the University of Pennsylvania for a mini-symposium with Dr. Cynthia Sung's origami robotics REU. Each MathILy-EST group gave a formal presentation to Dr. Sung's students, and vice-versa. During the last week of MathILy, the REU students gave a Daily Gather on their research work to the MathILy students. The two MathILy-EST groups will be presenting their results at the Joint Mathematics Meetings in Denver, CO, January 14–18, 2020. The coloring group will give one talk and the origami flip graph group will

give two talks (one on their connectivity result and one on the all-equal-angles case). These will be 10-minute talks in AMS general contributed paper sessions.

Writing: The MathILy-EST students each wrote daily about their research progress using Erik Demaine's Coauthor software. In addition, the students were given writing assignments and feedback by the Director. Each group prepared a draft of the final papers using Overleaf, with the eventual goal of having it placed on the arXiv and submitted for publication. Participants were also trained on writing CVs and tasked with creating their own professional CVs.

Software: Throughout MathILy-EST the participants used various software to aid in their research explorations. They used *Mathematica*, Sage, and Geogebra, wrote Python code to test and run algorithms, and used LaTeX to write up results and Illustrator to draw pictures.

Other professional development: The Director also provided training to the MathILy-EST students on giving good math presentations, looking up articles on MathSciNet or other databases, choosing graduate schools in mathematics, examining ethics in mathematics and academic culture (including the NSF's required "Responsible Conduct of Research" training), and deciding where they should give presentations on their summer work (e.g., the Joint Meetings and/or local MAA Section meetings).

Extracurriculars

Interactions with MathILy: The MathILy-EST participants lived in the same dorm as and ate meals with the MathILy summer program for high ability high-school students. While at first the REU students kept to themselves, eventually they played games and sports with the high-school students, took week-long classes with them during MathILy's Week of Chaos, and taught some of the MathILy students how to fold ninja throwing stars. The MathILy-EST students also attended all of the MathILy Daily Gathers and Life Seminars. The REU students were an integral part of the "choosing a college" Life Seminar, where they provided a range of perspectives since they came from a variety of institutions (large technical schools, large universities, and small liberal arts colleges).

Origami: One MathILy-EST student folded 1000 cranes, and other REU students helped hir try to string them in the traditional fashion. Also popular for folding were turtles, ninja stars, and origami dinosaurs. The dinosaurs were used to design the front of the MathILy-EST t-shirt. (The back of the t-shirt was designed by the Director using one of the origami flip graphs that the students discovered.) Modular Sonobé and PHiZZ units were also popular during the program, and collectively the participants made a large Sonobé icosahedron with each face triangulated further and a PHiZZ soccer ball.

Administrative matters

Facilities and staff at Bryn Mawr: The physical environment that Bryn Mawr provided for the MathILy-EST students was excellent. Two modern "fish bowl" style conference rooms in the newly renovated section of the Park Science Building were provided for exclusive use by the REU students. Each room had floor-to-ceiling blackboards and whiteboards and a wide-screen TV for projecting computer displays. Every time a need was expressed to the Bryn Mawr staff, they addressed it promptly (such as providing HDMI cables to use with the wide-screen TVs).

Post-Processing

Post-program senior personnel meeting: After the program a video conference was held between the MathILy and MathILy-EST Directors and three of the Senior Personnel on the NSF grant. This served to de-brief everyone on the first MathILy-EST summer as well as gave an opportunity for future MathILy-EST Directors to ask questions.

Impact: According to data from entrance and exit surveys, the MathILy-EST program increased participant interest in pursuing math graduate school and careers. In conversations with the Director during the final week of the program, the participants indicated excitement at taking math classes in the future that are related to the REU topic, such as graph theory and combinatorial geometry. The participants, in the research they produced this summer, made valuable contributions to the theory of flat-foldable materials that will lead to further research. The impact of MathILy-EST on the MathILy 2019 program was positive; when surveyed, MathILy students felt that MathILy-EST was valuable to their MathILy experience, both in general and on every specific aspect queried.

Finances summary:

The income from grant NSF DMS-1851842 was $51,555.

Total MathILy-EST income: $51,555.

There were no separate administrative expenses.

Total stipends (director, participants) were $28,330.

Total wages (MathILy director and Minion) were $2,630.

Total non-wage employee expenses were $305.

Total participant travel expenses were $717.

Program expenses (t-shirts, program outings) were approximately $209.

Site (Bryn Mawr College) charges, including housing and meals were $19,877.

Total MathILy-EST expenses: $52,068.

The approximately $500 in overage comes entirely from expenses that are not covered by NSF funding.