



Preface

The MathILy-EST Research Experience for Undergraduates (REU) was created to serve college-age students that are early in their college career (i.e., an emphasis on freshmen, but with consideration for sophomores and even graduating high-school seniors). Also, this REU runs in parallel to the 5-week MathILy program for talented high-school students. Theoretically, both programs take place at Bryn Mawr College and all the students and staff sharing the same dorm building, but in 2021, like 2020, both programs were conducted remotely online.

Program Preparations

Promotions

Emails: Notes advertising MathILy-EST were sent to the {MathILy, MathILy-Er} mailing list, to multiple email lists in the professional math community, and to (an updated list of) contacts at Historically Black Colleges and Universities and Minority Serving Institutions and the Hispanic Association of Colleges and Universities. sarah-marie was told about and then added to a national list of REU directors, whom she contacted.

Fliers: No fliers were sent this year because most events were canceled because of the COVID-19 pandemic.

Webpages and links: MathILy-EST has its own webpages, and is listed on the NSF-REU pages (of course) as well as the AMS Opportunities pages, the Institute for Broadening Participation's pathwaystoscience.org, the Math Alliance website, and the Art of Problem Solving's wiki. There are also several online lists of math REUs that include MathILy-EST.

Website traffic: There were about 10,000 impressions for the mathilyest/index.html page over the 2020–2021 season.

Other Activities: sarah-marie held an AoPS Math Jam on REUs in general and MathILy-EST in particular on March 2, with about 80 people during much of it and 450 people entering/leaving over the hour-and-a-half.

Applications and Admissions

Demographics: There were 80 completed applications for the 6 REU slots. Applicants originated from 26 US states (about 31 came from MA, CA, MN and NC). The small (for an REU) number of applications is almost certainly due to COVID-19.

The data in the following table was mostly self-reported by the applicants.

Stage in application	Female	Asian-American	African-American	Latinx	SLAC
All applicants	34%	18%	2.5%	4%	29%
1st cut of 28	57%	14%	7%	11%	36%
2nd cut of 22	59%	13%	9%	14%	45%
Shortlist (15)	53%	20%	0%	7%	47%
Accepted	67%	16%	0%	16%	50%

Every student invited to participate accepted, and all within 24 hours of invitation.

Personnel

Administrative: The MathILy-EST 2021 Director was Dr. Max Engelstein (mathematics faculty at the University of Minnesota). The PI on the NSF grant was sarah-marie belcastro (President of Mathematical Staircase, Inc.). The {MathILy, MathILy-Er, MathILy-EST} Minion was Madison Stuart.

Senior Personnel: These individuals gave advice on the construction of MathILy-EST and the NSF proposal for the grant that funds the program.

Hannah Alpert, mathematics faculty at Auburn University (MathILy-EST director 2020)

Nate Harman, Postdoctoral Assistant Professor at the University of Michigan

Thomas Hull, mathematics faculty at Western New England U. (MathILy-EST director 2019)

Peter Tingley, mathematics faculty at Loyola University Chicago

What Happened at MathILy-EST 2021?

Academics/Research

The research project was about discrete free boundary problems. It involved a mix of combinatorial, graph theoretic and analytical ideas, related to the Director's current work on continuous analogues of the same free boundary problem.

Reading: The first week of the program was spent on a reading assignment. The Director chose three research papers for students to read, split the students up into pairs and assigned one paper to each pair of students. Each paper contained relevant background information for the summer's research project. At the end of the week each pair of students gave a presentation to the other students.

Mathematical Explorations: Given any function on the vertex set of a (locally finite) graph, we studied the "energy" of that function (this energy functional was defined in a way that was both physically motivated and so that it would be a discrete analogue of a very important continuous energy). We wanted to understand which functions minimized this energy under the constraint that the function in question must take certain values on certain prescribed vertices. Without getting too detailed, the hope

was to understand how the geometry of the graph and the prescribed points could lead to different types of (both qualitative and quantitative) behavior of the minimizing function.

After the first week the Director posted a list of possible questions and avenues of exploration on Coauthor. Most of Weeks 2 and 3 were focused on exploring different aspects of different problems on this list without very much focus. The Director kept things intentionally open ended, partly because the problem had not been explored much and partly because he was uncertain which parts would turn out to be tractable.

By Week 4 the explorations had become more focused and were split into three main directions: (a) ensuring the existence of minimizers on graph and proving general quantitative bounds on their behavior; (b) categorizing/constructing minimizers, either explicitly or algorithmically; (c) questions of whether minimizers were unique, given a graph and prescribed vertex set.

Much progress was made on all of these avenues, and by Week 7 the students turned mainly towards writing their paper, which occupied the majority of their time in Weeks 7 and 8. By the end of the REU there was a solid rough draft of an arXiv preprint which the students wanted to polish a little more before sending to the Director.

Writing (during and after): At the beginning of the program writing took place mostly in the form of Coauthor posts. Initially, several members of the group did not feel particularly comfortable with TeX and mostly uploaded their handwritten notes into Coauthor. However, by the end all students seemed at least proficient with TeX.

Around the mid-program mark there was a concerted effort to turn the “stream of consciousness” Coauthor posts into good and rigorous proofs; the director asked each student to write up one proof “rigorously” and gave feedback on this work. Giving precise definitions was particularly difficult.

An Overleaf document was started sometime in Week 5. Students put their “rough ideas” into Coauthor and more polished proofs into the overleaf. Several days were spent deciding on the structure/narrative of the file and assigning different sections to different students, leaving the introduction for the end (as a mostly collaborative effort). By the end of the program all of the definitions, theorems and lemmas were in the document, though there was (and is) still some editing and polishing to be done.

Presentations: The MathILy-EST students made several presentations as a group. Every time someone visited MathILy to give a Daily Gather, the visitor was asked to spend time with the MathILy-EST students to hear a presentation on their research. These presentations were mostly seminar style (first on co-author then on Beamer) but often with lots of questions from the visitor(s). The MathILy-EST students found it very helpful to hear the thought processes of the visitors and also to organize their own thoughts about the project every time they gave a presentation.

During the last week of MathILy, the MathILy-EST students gave a Daily Gather on finding minimizers for path graphs—which was a central aspect of their research during the summer. The MathILy-EST participants seemed to be excited to engage in the trademark MathILy levity and prepared an elaborate backstory and illustrations to go along with their presentation. Rehearsal took quite a bit of time but the presentation seemed to go pretty well as a result.

In the last week of MathILy-EST (and the weeks after) the students prepared their abstracts for the JMM. They will be giving three 10-minute contributed talks, all of them joint. All of the students except one (whose schedule will not allow it) plan on (co-)giving presentations at the JMM.

Professional Development

Software: Students learned a lot about LaTeX skills, such as how to use Beamer and BibTeX. Most of students' LaTeX work was done using Overleaf. They also found an online graphics tool for making precise figures/calculations. They made daily use of Coauthor, Erik Demaine's open-source discussion board software for mathematical collaborations. Students also used software already familiar to them, including writing code in Python.

Other professional development: The MathILy-EST students were guided to write their own professional CVs, and heard about graduate school in two different sessions, one of which was a panel discussion by the MathILy staff who had been to graduate school in math. There were also professional development seminars on writing papers, giving presentations, how to use MathSciNet/ArXiv to look up journal articles, ethics in research and mathematics, and math opportunities for undergrads. MathILy-EST participants attended MathILy Life Seminars.

The Online Environment

Software tools: MathILy-EST met on Zoom, using the MathILy Zoom subscription. Scheduled 1–2 hour meetings occurred every morning with unstructured work time in the afternoon. Several Zoom rooms were left “open” all day so that the students could drop in and out for collaboration amongst themselves. Students used the Zoom share screen, whiteboard and annotate functions on Zoom but reported missing in person interaction. MathILy-EST was also part of the MathILy Slack workspace.

Social activities: The MathILy-EST students had several bonding activities during the week (and occasionally on weekends) that they planned themselves, including online games and lunch hours.

Post-Processing

Post-program senior personnel meeting: Near the end of the program, there was a meeting to discuss the program and to plan for next year. Nate Harman agreed to direct the REU in 2022.

Impact: Students rated almost all aspects of their MathILy-EST experience as very or somewhat valuable, and all six participants stated that participation in MathILy-EST has positively influenced their career path or career. At the start of the program, most of the students thought there were at least a few career opportunities in the mathematical sciences and a couple of them thought there were a lot of opportunities; by the end almost all students had shifted to believing there were a lot of different opportunities available, with everyone believing there were at least a few. About three-quarters of the MathILy students found MathILy-EST valuable to their MathILy experience in general, with a bit over half giving this rating on each specific aspect queried.

Finances summary:

The income from grant NSF DMS-1851842 was \$32,871.

Total MathILy-EST income: \$32,871.

There were no separate administrative expenses.

Total stipends (director, participants) were \$28,330.

Total wages (MathILy director and Minion) were \$1,841.

Total non-wage employee expenses were \$141.

Program expenses (t-shirts) were approximately \$162.

Travel expenses for participants were \$300.

Meal stipends were \$2,400.

Total MathILy-EST expenses: \$33,174.

The approximately \$303 in overage comes entirely from expenses not covered by NSF funding.

Note that there is still travel money in the grant budget that will be used to help students defray costs of attending the Joint Mathematics Meetings in January 2022.