



MathILy 2017 Final Report

Preface

MathILy was about the same size in its fifth year (*fifth year!*) as in its fourth year. As usual, we held a program with excellent students who learned a lot of mathematics and a lot about how to think and speak and write mathematically.

Program Preparations

Promotions

Electronic: Emails were sent to promising applicants from 2016 and to MathILy 2016 participants. MathILy continues to be listed on several high-traffic webpages, including the summer program listing pages hosted by the AMS, MIT Admissions, and the Art of Problem Solving Wiki. Web traffic tripled over the last year, with a substantial amount of the traffic from abroad.

Print: About 2600 fliers with basic information on one side and a multi-part mathematics problem on the reverse were distributed at national and local mathematics contests and enrichment activities. Fliers were handed out by humans at HMMT (February) and ARML Penn State.

Other Activities: At HMMT, sarah-marie gave a Mini-Event (on coverings and matchings) and was available to talk with students, parents, and coaches. She also offered a Math Jam (on multibackwards numbers) through the Art of Problem Solving that was followed by a {MathILy, MathILy-Er} Q&A, and gave the Friday Night Lecture (on counting faces in n -cubes and n -simplices) at ARML Penn State, before and after which she met with potential applicants, admitted students, and alumns.

Applications

Statistics: We received 635 Short Forms, 218 Not-as-Short Forms, 200 EARs, and 184 completed applications. We admitted 46 students. This reflects an admissions rate of about 25%, which is low. Six students declined for other national-level summer mathematics programs.

Demographics: Not-as-Short Forms originated from at least 24 US states and 18 foreign countries (representing mostly North America, Europe and Asia, but also including South America and Oceania).

The data in the following table was measured where possible and approximated otherwise.

Percentage	Female	East Asian	South Asian	Latin@	Middle Eastern	Other of color
Short Forms	37%	39%	17%	3%	3%	4%
EARs	31%	43%	13%	1%	1%	3%
Attending	24%	46%	10%	3%	0%	3%

Financial Aid: We awarded \$9800 in financial aid to MathILy participants, of which \$7000 was provided by the AMS Epsilon Fund and \$1500 was provided by individual donors. We met the level of demonstrated need for all admitted students. Surprisingly few students applied for financial aid, which is why our total amount awarded was less than half of the amount awarded for 2016.

Personnel

Academic: Lead Instructors were Dr. Hannah Alpert (Ohio State U., Ph.D. MIT 2016), dr. sarah-marie belcastro (Smith College Research Associate, Ph.D. University of Michigan 1997), Dr. Thomas C. Hull (Western New England U., Ph.D. University of Rhode Island 1997), Dr. Emily Peters (Loyola University Chicago, Ph.D. University of California at Berkeley 2009), and Dr. Daniel Studenmund (University of Notre Dame, Ph.D. University of Chicago 2014).

Apprentice Instructors were Brian Freidin (graduate student, Brown University), Joshua Munding (undergraduate student, Swarthmore College and MathILy 2013 alumn), Jason Saied (graduate student, Rutgers University), and Corrine Yap (graduate student, Rutgers University).

Biographical information and prior experience are listed at <http://mathily.org/dramper.html>.

Administrative: The Director was dr. sarah-marie belcastro. The excellent {MathILy, MathILy-Er} Minion was Madison Stuart (Smith College B.A. 2006 in Mathematics and German; graduate work in information science at the University of Michigan). The Protector and Responder in the MathILy Environment (PRiME) was Sara Canilang (Carleton College undergrad Math and Religion major)..

Advisory Amalgam: These individuals gave advice on academic and practical aspects of MathILy.

[Dr. Douglas J. Shaw](#), mathematics faculty at University of Northern Iowa

[Dr. Ruth Haas](#), mathematics faculty at University of Hawaii

[James Cocoros](#), mathematics faculty at Stuyvesant High School

[Dr. Dylan Shepardson](#), mathematics faculty at Mount Holyoke College

[Dr. Carol E. Fan](#), operations researcher (currently Head of Data at [Blackbird](#))

[Dan Zaharopol](#), Executive Director of [BEAM](#)

[Dr. James Tanton](#), mathematician, currently Ambassador for the [MAA](#)

[Dr. Joshua Greene](#), mathematics faculty at Boston College

[Dr. Emily Peters](#), mathematics faculty at Loyola University Chicago

[Wing L. Mui](#), mathematics faculty at the Overlake School

[Dr. Thomas Hull](#), mathematics faculty at Western New England University

[Dr. Josh Laison](#), mathematics faculty at Willamette University

Student Demographics:

States represented by MathILy students, roughly from east to west: Rhode Island, Massachusetts, New York, New Jersey, Pennsylvania, Maryland, Virginia, North Carolina, Florida, Indiana, Illinois, California, and Washington.

Countries outside of the United States represented by MathILy students, roughly from east to west: China, Switzerland, Italy, Canada, and Brazil.

Gender breakdown: 9 females, 28 males.

Ages: One 14-year old; five 15-year olds; eighteen 16-year olds; twelve 17-year olds; and one 18-year old.

Academic backgrounds: A bit under half of the students had taken calculus II or equivalent (and 18% had also taken multivariable calculus), 4 had taken linear algebra, 3 had taken abstract algebra, and 2 had done independent studies in topology. 15 students had attended summer mathematics programs before.

What Happened at MathILy 2017?

Academics

Classes: Each weekday we had 4 hours of morning class, 1–1.5 hours of Daily Gather, and 3 hours of evening class, for at least 8 contact hours per day (not counting mathematical conversations outside of class). Weekends varied, but most Saturdays included 4 hours of morning class.

The basic curricular structure was two weeks of core curriculum, called Root Class (after the root of a graph theoretic tree, and after the idea that the material strengthens student grounding much as the roots of a tree do), followed by one week of short topical classes, called Week of Chaos, followed by two weeks of focused-topic classes, called Branch Class (after branches of mathematics, and after the idea that tree branches grow from a strong trunk nourished by roots).

Root Class: There were three Root classes, each with 12 students, one taught by {sarah-marie, Jason}, one taught by {Hannah, Corrine, Josh}, and one taught by {Daniel, Brian}. Our core curriculum consisted of linear and affine algebra and geometry (including equations and intersections of hyperplanes, span, linear independence, transformations, and dimension), combinatorics, graph theory, definition and examples of groups, isomorphism for various categories, probability spaces and expected value, and basic cardinality. Of course, all of this material was treated with full proofs given by the students. Our only returning student worked through much of, and highly enjoyed, the advanced linear algebra curriculum.

Week of Chaos: Students indicated which of 51 potential topics they would be excited to learn about, from which instructors decided on a list of classes offered. These were Eigenonion (eigenvalues/vectors), Geometric Classification (moduli spaces), Moar Combinatorics, Big Finite Diff, Projective Geometry, How to Treat TMM Syndrome (Markov chains), The Algebra Safari (group theory), Generatingfunctionology, How to Play Poker (probability on simple games), Loops on Loops on Loops (homotopy theory), Complex Analysis is the Real Deal, The Secret Course (cryptography), Voting Methods—or why we can't have nice things: a proof, The Characters of Groups (representation theory) Le Baguette Theory (Lebesgue integration), Erdős Magic (the probabilistic method), p-sychie p-phenomena (p-adics), Non-Euclidean geometry, Knot Theory—Practice, Math Saves the World: Combinatorial Optimization, Surreal Numbers: A Play in Five Acts, Groups and Graphs (Cayley graphs), Serious Study of a Pastime (combinatorial game theory), Ramen Theory (Ramsey theory), and More Marxist Mayhem (spectral theory). Each student was placed in 5 classes according to expressed preferences. Approximately $2/3$ of these classes used specific material from the Root curriculum, and approximately $1/3$ benefitted substantially from students' knowledge/understanding of linear algebra.

Branch Classes: We offered three Branch classes, one on topological graph theory (sarah-marie, Josh, and Corrine), one on the mathematics of folding flat materials (Tom and Brian), and one on discrete and continuous computation (Hannah and Jason). All three Branch classes used linear algebra, and all three introduced real-world applications.

Pedagogy: All classes were conducted using inquiry-based learning, with the bulk of the time spent with students working in groups or presenting their insights to each other and a much smaller amount of time used by faculty conducting discussion from the board.

Feedback: Students received feedback in multiple ways. They received instant verbal feedback on the correctness of their mathematical ideas during class, as well as feedback on use of notation, language,

and presentation style. They received written feedback on the problem solutions and proofs they handed in each day, always within 24 hours of handing in work. Near the end of Root and Branch classes, each student was asked to write an introspective self-evaluation. The self-evaluations were discussed by the student's instructors, and the instructors then held a 5–20 minute meeting with each student to give overall feedback on the student's progress at MathILy and advice for the future.

Daily Gathers: Each instructor gave a Daily Gather. The Daily Gather timeslot was also used to show Math Movies once per week. These included expository films made by the Mathematical Association for America's Geometry Project, films from the National Film Board of Canada, and narrated animations made by individual mathematicians or research groups. The remainder of the Daily Gathers were given by visiting mathematicians, both local (Swarthmore and Lebanon Valley Colleges, and Lehigh and Villanova Universities) and from Illinois, Maryland, Massachusetts, Texas, and Vermont.

Extracurriculars

Life Seminars: There were three weekend Life Seminars offered. The first was on practical matters such as how to address faculty in person or by email and how to do laundry. The second Life Seminar was on careers for people with mathematical science training, with a section on the college application process. The third Life Seminar was on how to choose colleges to which to apply, and included an exchange of information about instructor experiences at several colleges and universities.

Other all-program activities: At the end of the first week, we walked over to Haverford College, had a mini Daily Gather in their math lounge, and returned to Bryn Mawr. We then gathered in the usual Daily Gather room to play a rousing game of telephone pictionary. The program went by train to Philadelphia for the entire Saturday right after Week of Chaos. The next day, the program was treated to ice-cream cones and dishes at a local shop on National Ice Cream Day.

Non-program-wide activities: Throughout the program, chess was popular and two boards were set up and frequently in use in our common area. Students irregularly played Magic and/or Dominion. Late on many nights, students gathered to practice playing guitar and singing badly in a particular student's room. Lots of students went swimming in the afternoons. Frisbee-playing was infrequent. A few students went running regularly, and one ran over to Haverford, used their outdoor track, and ran back. We had two sessions in the Dance Studio, each involving about 2/3 of the program. Students and staff used the piano in our dorm a lot once its hammers appeared. We also had access to a music practice room that was frequently and regularly used. All but four students attended a semi-impromptu music and dance talent show one Saturday night. Most nights there were optional Bedtime Stories. A group of geese who regularly appeared on Merion Green were dubbed GeesIGy. The week a magic camp was on campus, students benefitted from multiple impromptu magic performances and lessons by their participants. Twice a small dog seemed to teleport into one of the Branch classes. On the last night of the program, we brought lots of puzzles to the common area and many of the students played with them late into the night and again on departure day.

Administrative matters

Facilities at Bryn Mawr: Again, everyone liked the facilities and the campus as a whole. We were in the same part of the same dormitory as most prior years, and allocated parts of two floors. Some students had single rooms, and others shared one- or two-room doubles or three-room triples (each with at least one window air-conditioning unit). We were assigned our choice of classrooms in Park Science Center. Janitorial service was extensive; every night chalkboards were cleaned and the rooms were tidied.

Campus Location: Bryn Mawr is a safe and tree-filled suburb of Philadelphia, located on a major train line into the city. Bryn Mawr College is a few blocks away from the Bryn Mawr train station, and also a few blocks away from a major street that parallels the train line and has lots of shops, libraries, post offices, etc. This location was convenient for our purposes—many desired destinations were a short walk away or required a straightforward and quick drive. Students were able to depart campus on foot in groups to go out for dinner or shopping. It was also straightforward to take the program into the city, and use of the train made our travel flexible.

Post-Processing

Post-program staff meeting: After the students left, the staff convened to evaluate various aspects of the program and to discuss how we could improve the workings of MathILy in future.

All agreed that the pre-program teaching-related documents and training were helpful. It was suggested that we create a video showing some typical snippets of Root class to show to new instructors at the pre-program training, which seems like an obviously useful thing to do. We took a video sample this year and plan to augment or replace it next year. Three instructors worked on creating a short AI Guide that summarizes logistics and norms that new AIs might find useful. A past PRiME made substantial revisions and additions to the PRiME Guide. In order to make it easier for students to interact with Daily Gather visitors outside of the Daily Gather time, we decided that in future we would ask pairs of students to host visitors.

Instructors collectively felt that the students had enough more background than prior cohorts that even the weakest students were able to follow along easily. Despite a lot of student passivity we were satisfied with student growth. In discussion instructors named at least a third of the students as having made significant progress, including some of the strongest and some of the weakest students.

Impact: As usual, several students commented

- that they learned about areas of math of which they hadn't even dreamed before MathILy
- that they are now certain they want to pursue more mathematics.

Finances summary:

The income from student fees (some discounted) was \$157,427.

Donations earmarked for financial aid were \$1,500.

Our Epsilon Grant award was \$7,000.

Total MathILy income: \$165,927.

Administrative expenses (insurance, fliers, etc.) totaled approximately \$2239.

Total wages (instructors, PRiME, Minion, Director) were approximately \$53,481.

Travel costs (Daily Gather speakers, instructors) were approximately \$2253.

Program expenses (supplies, program outings) were approximately \$2383.

Site (Bryn Mawr College) charges, including housing, meals, and duplications were \$93,600.

Total MathILy expenses: approximately \$153,956.

An externally administered travel grant paid for an additional \$2180 in travel expenses. We were also fortunate to receive in-kind donations of volunteer time and travel expenses, worth roughly \$7,634, about \$35 worth of linens, and software from Wolfram Research, valued at \$5412.

The bulk of the net revenue arose from significant unexpected budget underruns. Mathematical Staircase, Inc. generates no non-program income, so MathILy surplus revenue pays for administrative expenses such as regulatory required financial review and state corporate filings. Additionally, we expect to be able to keep our fees the same for a fifth year (*fifth year!*).