## MathILy 2023 Final Report

## Preface

MathILy had more participants than ever before, and was $50 \%$ female and non-binary. As usual, we held a program with excellent students who learned a lot of mathematics and a lot about how to think and speak and write mathematically.

## Program Preparations

## Promotions

Electronic: Individual emails were sent to prior participants and promising applicants. MathILy continues to be listed on several high-traffic webpages, including MIT Admissions' "Preparing for MIT" summer programs page. Web traffic varied from 6,500-15,000 hits/month, with about $35 \%$ of the traffic from abroad. The second-most popular page (after the MathILy home page) is Discrete Mathematics in the Real World, accounting for about $20 \%$ of website visits.

Print: We sent 100 fliers to the Harvard-MIT Math Tournament (HMMT) with sarah-marie, and 350 to the two contests that responded to the Minion's queries.

Other Activities: We held a \{MathILy, MathILy-Er, MathILy-EST\} Yearly Gather at the Joint Mathematics Meetings (online, April), where at least 85 participants team-solved a Jonah-Ostroff-designed three-part puzzle themed around the three programs (with a meta for those who finished all three). At HMMT February online, sarah-marie gave an Education Event on Drawing Networks on Doughnuts, with about 80 attendees. In late March she offered an Art of Problem Solving Math Jam (about 78 participants) on Views of the $n$-cube followed by a $\{$ MathILy, MathILy-Er $\}$ Q\&A.

## Applications and Admissions

Statistics: We received 1690 Short Forms, 644 Not-as-Short Forms, 614 EARs, and 550 completed applications. We admitted 59 students, for an admissions rate of $\sim 11 \%$. Thirteen admitted students declined, eight for four other summer math programs. Thus, our current yield rate is roughly $78 \%$.

Demographics: Applicants originated from at least 39 US states/territories/districts and 31 foreign countries (representing North/Central/South America, Europe, Asia, and the Middle East). The data in the following table was measured where possible and approximated otherwise; the final row reflects a post-program demographic survey given as part of assessment for the MathILy-EST NSF grant.

| Percentage | Female | NB | East Asian | South Asian | Latinx | Middle Eastern | Black, Indigenous |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Short Forms | $33 \%$ | $1 \%$ | $42 \%$ | $24 \%$ | $5 \%$ | $9 \%$ | $1 \%$ |
| EARs | $33 \%$ | $2 \%$ | $57 \%$ | $15 \%$ | $3 \%$ | $3 \%$ | $1 \%$ |
| Attending | $48 \%$ | $2 \%$ | $50 \%$ | $9 \%$ | $4 \%$ | $0 \%$ | $0 \%$ |

Financial Aid: We awarded $\$ 13,675$ in financial aid to MathILy participants (\$9,975 to domestic students and $\$ 3,700$ to international students), and used grants from Jane Street $(\$ 3,000)$ and from the AMS Epsilon Fund $(\$ 5,000)$ for this purpose. Of admitted students, $15 \%$ applied for financial aid; we met the demonstrated need of all applicants.

## Personnel

Academic: Lead Instructors were dr. sarah-marie belcastro (Math Staircase Inc., Ph.D. U. of Michigan 1997), Dr. Brian Freidin (Auburn U., Ph.D. Brown U. 2018), Dr. Thomas C. Hull (Franklin\&Marshall C., Ph.D. Univ. of Rhode Island 1997), Dr. Emily Peters (Loyola U. Chicago, Ph.D. UC-Berkeley 2009), and Dr. Daniel Studenmund (Binghamton U., Ph.D. U. of Chicago 2014). Apprentice Instructors were David Gonzalez (graduate student at UC-Berkeley, MathILy 2014), Nadav Kohen (graduate student at Indiana U., MathILy 2015), Frank Lu (entering graduate student at Harvard U., MathILy 2018), Kye Shi (graduate student at UCLA, MathILy 2015/2016), and Natasha Ter-Saakov (graduate student at Rutgers U., MathILy 2014/2015). Biographical information and prior experience are listed at Dramatis Personae.

Administrative: The Director was dr. sarah-marie belcastro. The excellent \{MathILy, MathILy-Er\} Minion was Madison Stuart (Smith College B.A. 2006 in math and German; graduate work in information science at the University of Michigan). The PRiME FACToRs (Protectors and Responders in the MathILy Environment and Facilitators of Activities and CriTiquers of wRiting) were Basia Klos (entering graduate student at CU-Boulder), Jessica Lee (undergraduate at Harvard C., MathILy 2018,/2019), and Thea Rugg (entering graduate student at Cornell U.). The PRiME FACToRs had academic roles as well.

Advisory Amalgam: These individuals gave advice on academic and practical aspects of MathILy.
Dr. Douglas J. Shaw, mathematics faculty at University of Northern Iowa
Dr. Ruth Haas, (retired) mathematics faculty at University of Hawaii
James Cocoros, mathematics faculty at Hunter High School
Dr. Dylan Shepardson, mathematics faculty at Mount Holyoke College
Dr. Carol E. Fan, operations researcher (currently Operations Data Science Lead at Apple)
Dan Zaharopol, Executive Director of BEAM
Dr. James Tanton, mathematician, currently Ambassador for the MAA
Dr. Joshua Greene, mathematics faculty at Boston College
Dr. Emily Peters, mathematics faculty at Loyola University Chicago
Wing L. Mui, Seattle-area artist and former mathematics teacher
Dr. Thomas Hull, mathematics faculty at Franklin\&Marshall
Dr. Josh Laison, mathematics faculty at Willamette University

## Student Demographics

U.S. States represented by MathILy students, roughly from east to west: Massachusetts, Connecticut, New York, New Jersey, Maryland, Pennsylvania, Florida, Virginia, North Carolina, Georgia, Michigan, Illinois, Minnesota, Colorado, Wyoming, Washington, and California.
Countries outside of the U.S., roughly from east to west: Canada, Mexico, South Korea, China, Ukraine, Spain. Gender breakdown: There were 22 female, 1 nonbinary, and 23 male participants.
Ages: There were two 14 -year olds, seventeen 15-year olds, twelve 16-year olds, thirteen 17-year olds, and two 18-year-olds.

Academic backgrounds: Half of the students had taken calculus II (and 17\% had also taken multivariable calculus), and four had taken linear algebra. In contrast, $11 \%$ of the students had not yet taken precalculus. Seventeen students had attended summer mathematics programs before.

## What Happened at MathILy 2023?

## Academics

Classes: Each weekday we had 4 hours of morning class, 1-1.5 hours of Daily Gather, and 3 hours of evening class, for at least 8 contact hours per day (not counting mathematical conversations outside of class). Weekends were, as always, a bit idiosyncratic, but the general Saturday template consisted of 4 hours of morning class and 2 hours of afternoon Life Seminar.

The basic curricular structure was two weeks of core curriculum, called Root Class (after the root of a graph theoretic tree, and after the idea that the material strengthens student grounding much as the roots of a tree do), followed by one week of short topical classes, called Week of Chaos, followed by two weeks of focusedtopic classes, called Branch Class (after branches of mathematics, and after the idea that tree branches grow from a strong trunk nourished by roots).

Root Class: There were three Root classes, each with 15 or 16 students, one taught by \{sarah-marie, Kye, Jessica\}, one taught by \{Daniel, David, Basia\}, and one taught by \{Tom, Frank, Thea\}. Our core curriculum consisted of linear and affine algebra and geometry (including equations and intersections of hyperplanes, span, linear independence, transformations, and dimension), combinatorics, graph theory, examples of groups, isomorphism for various categories, probability spaces and expected value, and basic cardinality. Of course, students gave full proofs for all of this material.

Week of Chaos: Students indicated which of 58 potential topics they would be excited to learn about, from which instructors decided on a list of 26 classes offered. These were: Through the Looking Glass (random walks), Origametry, How to deliver things . . . kinda (network flows), Geometric Library Science (moduli spaces of polygons), Geometry, but more different (non-Euclidean geometry), Why We Can't Have Nice Things (voting theory), Ramsey theory, Through Thicc and Thinn (fractal geometry), Top Secret d1cb1287b3b64a36854069f8313394ae53934b04e56b20e9a6cfb290228f4593 (cryptography), The Geometry of Mordor (projective geometry), Sneaky Proofs (zero-knowledge proofs), Game Theory, Math about Math (mathematical logic), Mathematics of Knitting, $p$-adics and You!, Math Saves the World: Combinatorial Optimization, Banana Measure (Lebesgue integration), Loopy loops (homotopy theory), Algebraic Geometry, What next, what before, and what do? (Markov chains), Mandy and the Julias (Mandelbrot set), The Secrets of $\mathbf{Z}_{p}$ (quadratic reciprocity), Groups and Graphs (geometric group theory), it's möbin time (Möbius tranformations), Arguing about Aligned Alpacas (complexity theory), and Really Hard Calculus (complex analysis). Student preferences guided placement of each student into 5 classes. More than $3 / 5$ of these classes used specific material from the Root curriculum, nearly $1 / 3$ benefitted substantially from students' knowledge/understanding of linear algebra, and nearly $1 / 3$ used technology in a significant way.

Branch Classes: The Branch classes were on topological graph theory (sarah-marie and David, themed around snails eavesdropping on garden gossip), on topological algebra (Emily and Nadav and Frank, themed around food), and on discrete geometry (Tom and Kye with Jessica, themed around dinosaurs). All three Branch classes used linear algebra, all three used computer algebra systems, and all three of them introduced real-world applications.

Pedagogy: All classes were conducted using inquiry-based learning, with the bulk of the time spent with students working in groups or presenting their insights to each other and a much smaller amount of time used by faculty conducting discussion from the board. Students were assigned to take comprehensive notes for future class reference, and after instructor review (and revision) were copied and distributed to the class.

Feedback: Students received feedback in multiple ways. During class, they received instant verbal feedback on the correctness of their mathematical ideas, and also on use of notation, language, and presentation style. Likewise, students received daily written feedback on their mathematical writing. Near the end of Root and Branch classes, each student was asked to write an introspective self-evaluation. The self-evaluations were discussed by the student's instructors, and the instructors then held a 5-20 minute meeting with each student to give overall feedback on the student's progress at MathILy and advice for the future.

Interactions with MathILy-EST: MathILy-EST participants collaborated with MathILy students at Daily Gathers throughout the program. Additionally, each MathILy-EST participant took a Week of Chaos class.

Daily Gathers: Each instructor gave a Daily Gather, as did the MathILy-EST research group. We had seven external visitors. The Daily Gather timeslot was also used to show Math Movies once per week. These included expository films and narrated animations. The remainder of the Daily Gathers were given by visiting mathematicians. Each Daily Gather speaker provided some insight into that person's perspective on the mathematical enterprise and/or way of being a mathematician.

## Extracurriculars

Life Seminars: There were five weekend Life Seminars offered, many with time for open questions. The first was on practical matters such as how to address faculty in person or by email, and impostor syndrome. The second Life Seminar was on careers for people with mathematical science training. The third Life Seminar was about preparing for Branch, when to start research, and issues surrounding collection of demographic information. The penultimate Life Seminar was on how to choose colleges to which to apply, and included advice from MathILy-EST participants. The final Life Seminar, held the following afternoon, was on how to readjust to the non-MathILy world post-MathILy.

All-program activities: At the end of the first week, we walked over to tour Haverford College. The next day we had a rousing game of telephone pictionary. At the end of Week of Chaos we went to Philadelphia for the afternoon and celebrated National Ice Cream Day by treating the program to ice cream at a local shop.

Social activities: A table in the largest common area was commandeered for indoor ping-pong. A handful of girls went running most mornings before breakfast The card game Canadian Fish was popular and a low-key tournament was held over multiple weeks. On weeknights Bedtime Stories read from PopCo. A MathILyEST/MathILy string quartet (plus clarinet) formed, practiced, and gave a concert. A student's love of bananas became a theme, as did contests (haiku; meme) for seats on trips to Target. (The banana-favoring student ate 150 bananas over the five weeks.) The Sophia Club met with three student Sophias, one local Sophia, and one remote Sophia; a student-written voting form for superlatives included items "Most likely to be secretly a Sophia" and "Most likely to actually be a Sophia" (options: Sophia., Sophia!, Sophia?). A returning alumn posted an encrypted message on the bulletin board (reminiscent of the encrypted messages in PopCo) that was the first of four puzzles, where the decryption of each message riddled the location of the next message. On departure day, there were massive flight delays (and some cancellations) so students stuck in the airport posted on Slack, found each other for meetups, and posted meetup photos on Slack.

## Administrative matters

Pandemic precautions: We required rapid testing on arrival for all participants, masking everywhere for the first week, and then allowed unmasking in MathILy-only spaces on evidence of a second negative rapid test. We ate most meals outdoors, but when it rained we brought meals back to the dorm to eat. Students were not allowed to leave campus during MathILy except when accompanied by staff.

Campus Location: Bryn Mawr is a safe and tree-filled suburb of Philadelphia, located on a major train line into the city. Bryn Mawr College is a few blocks away from the Bryn Mawr train station, and also nearby shops that supply toiletries and foodstuffs. Actual grocery stores are a short drive away.

Facilities at Bryn Mawr: Again, everyone liked the facilities and the campus as a whole. We were assigned our own dormitory with air-conditioned rooms and some air-conditioned lounges. One of our three classrooms in Park Science Center had floor-to-ceiling blackboards. Janitorial service is extensive at Bryn Mawr; every (early) morning chalkboards were cleaned and the rooms were tidied. The dining hall staff went out of their way to be helpful to us, and made a variety of extra dishes exclusive to our program.

## Post-Processing

Post-program meetings: After the official end of the program, the staff convened to evaluate various aspects of the program and to discuss how we could improve the workings of MathILy in future. We will enhance AI training on critiquing and provide sample structured lesson plans. We are considering changes to the Philadelphia trip for when pandemic restrictions are no longer needed.

Impact: As usual, many students commented that they learned about completely new areas of math and did so in depth, that they are even more passionate about mathematics, and that their mathematical communication skills improved significantly. Additionally, several students were surprised and gratified by the integral role of proof making and writing at MathILy, and many noted that they really had no prior exposure to proofs.

## Finances summary:

The income from student fees (some discounted) was $\$ 214,037$.
Grant support (Jane Street, Epsilon, individual researchers) was \$46,940 [some awaiting approval]. Total MathILy income: \$260,977.

Administrative expenses (insurance, fliers, etc.) totaled approximately $\$ 2,957$.
Total wages (instructors, PRiME, Minion, Director) were approximately $\$ 81,428$.
Wage-related administrative costs (payroll taxes, etc.) were $\$ 3,369$.
Travel costs (visitors, instructors) were $\$ 6,483$.
Program expenses (supplies) were approximately $\$ 5372$.
Site expenses from Bryn Mawr were $\$ 147,787.21$.
One-year membership to MAA for interested alumns is $\$ 375$.
Total MathILy expenses: approximately $\$ 247,771$.
The individual researcher NSF grants supported subsistence, stipends, and travel for trainee staff members. Jane Street funded financial aid (as did the Epsilon Fund grant) and visitor travel. We were fortunate to receive donations of software from Wolfram Research worth $\$ 7830$, and volunteer time, travel, and housing worth roughly $\$ 1890$.

The net revenue of approximately $\$ 13,206$ arose primarily from from surprise NSF rebudgeting.

